

Charged Domain Walls

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Abstract

In this paper we investigate Charged Domain Walls (CDW's), topological defects that acquire surface charge density Q induced by fermion states localized on the walls. The presence of an electric and magnetic field on the walls is also discussed. We find a relation in which the value of the surface charge density Q is connected with the existence of such topological defects.

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1. Introduction

In the fundamental theories of elementary particles, the spontaneous symmetry breaking by the Higgs mechanism play a central role. Such theories in general have a degenerate vacuum manifold with a non-trivial topology. In the recent years a lot of investigations on topological defects have been done, with applications on different fields, as for instance particle physics or cosmology [1]. In this paper we study topological defects, called domain walls, that are inherent to field theories with spontaneously broken discrete symmetries. The scalar field has isolated minima and the walls are surfaces interpolating between minima of the scalar potential with different vacuum expectation values of the scalar field. This phenomenon, occurrence of the domain walls, is quite common in solid state physics. In cosmology, such structures can form by the Kibble mechanism [2], and therefore must be taken into account in cosmological considerations.

The idea of this work is connected with the so called Q-balls. These non topological solitons can be created during a phase transition [3]. In the mid-70's new topological solitons were considered in the context of systems with two scalar fields [4]. In the mid-80's they reappeared Coleman's works [5], and they were given the name of Q-balls. In the mid-90's, the Q-balls were connected with models of supersymmetry breaking, as in Ref. [6, 7], in which the Q-balls are condensates of squark or slepton particles. These condensates are connected in baryogenesis by the so-called Affleck-Dine mechanism [8]. Thus, the Q-balls might be important because they might contribute to the dark matter in the Universe [7, 9]. As a consequence, the people began to consider Q-balls and Q-strings (extended objects of the Q-balls type [10, 11]). The study of Q-walls and Q-strings is also connected with their lifetime [12], that is arbitrarily large. This is a very important point because these configurations might be of relevance in the formation of structures in the early Universe. Moreover, it has been considered domain walls carrying a $U(1)$ charge. These system are composed by two interacting scalar fields, the Higgs real field and a complex scalar field [13, 14].

It is important to consider two aspects in this analysis. The first is the interaction between particles and domain walls. This study start from the Voloshin's paper [15] and continues with the work of Ref. [16]. A recent analysis by the authors of Ref. [17–19], considers the scattering of fermions off domain walls at the electroweak phase transition in presence of a primordial magnetic field. The second paper deals with the trapping of the massless fermions in these defects. In fact it is well known that Dirac fermions with Yukawa coupling with scalar field develop zero mode solution near a domain wall [20, 21]. These zero mode behave like massless fermions in the two dimensional space of the wall. Recently we have analyzed the zero energy solutions localized on the wall in presence of a magnetic field on the wall. The localized states are a peculiar characteristic of the domain wall and they play an important role in the dynamics of the walls. The localizing of the fermions in the core of the defect is important for building our analysis in this paper in which we investigate charged domain walls (CDW), in which the charge is not due to the charged scalar fields, as in the Q-balls theory, but to the interaction with the massless fermions. In particular our analysis connects the charge Q with some parameters of the theory in order to have some constraint about the existence of such CDW's. The physical motivation to investigate CDW's was provided mainly through the works by Lopez [22] and Grøn [23] and the study of repulsive gravitational fields [24].

The organization of this paper is as follows. In the next section we study the equation

of motion of lagrangian density of a $\lambda\phi^4$ theory with the scalar field coupled by Yukawa coupling to massless fermions. In particular we find the wave function of localized zero mode solution. In Sec. III we also introduce the electromagnetic field and we find the general expression for the wave function interacting with domain wall. Moreover we analyze the magnetic and electric field near the wall. In Sec. IV we study the system formed by the domain wall and a fermionic plane wave, in order to compare the energy density of this system with the system in Sec. III. In Sec. V we conclude with some cosmological implications of these charged domain walls.

2. Fermionic zero-modes localized on the wall

In this Section we consider a single real self-interacting scalar field, coupled with a massless Dirac fermion through Yukawa coupling. The Lagrangian density of the system is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \bar{\psi}(i\cancel{\partial} - g_Y\phi)\psi, \quad V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2, \quad (2.1)$$

where the potential $V(\phi)$ has degenerate minima at $\phi = \pm\eta$. In this paper we restrict ourselves to the $(1+1)$ -dimensional case, and we suppose that

$$\phi = \phi(x), \quad (2.2)$$

$$\psi = \psi(x, t) = \xi(x) e^{-i\omega t}, \quad (2.3)$$

where ϕ and ξ are real functions of x . We will use the following representation of the Dirac matrices,

$$\gamma^0 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad (2.4)$$

where σ^i , $i = 1, 2, 3$, are the Pauli matrices. The Lagrangian (2.1) with Eqs. (2.2)-(2.4) implies the equations of motion

$$\phi'' + \lambda(\phi^2 - \eta^2)\phi = g_Y\bar{\xi}\xi, \quad (2.5)$$

$$i\gamma^1\xi' + (\omega\gamma^0 - g_Y\phi)\xi = 0. \quad (2.6)$$

(Here, and throughout, a prime will denote differentiation with respect to x .) In this paper we make the following ansatz for the spinor ξ ,

$$\xi = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ u \\ v \\ v \end{pmatrix}, \quad (2.7)$$

where u and v are real scalar functions of x . Because $\bar{\xi}\xi = 0$, Eq. (2.5) take the form

$$\phi'' - \lambda(\phi^2 - \eta^2)\phi = 0, \quad (2.8)$$

that is the equation of motion for $\lambda\phi^4$ theory. It is simple to see that Eq. (2.8) admits the solution describing the transition between two adjacent regions with different values of ϕ , that is $\phi = +\eta$ and $\phi = -\eta$. The profile takes the form

$$\phi(x) = \eta \tanh(x/\Delta), \quad (2.9)$$

where $\Delta = \sqrt{2/\lambda} \eta^{-1}$ is the thickness of the wall, and gives origin to the so called “domain wall” which is thought to be formed in a continuous phase transition by the Kibble mechanism [2]; the domain wall is the interpolating region of rapid change of the scalar field.

With the ansatz (2.7), Eq. (2.6) splits in two equations

$$u' + g_Y \phi u = 0, \quad (2.10)$$

$$v' - g_Y \phi v = 0, \quad (2.11)$$

with $\omega = 0$. The physical solution of Eq. (2.11) is $v = 0$, since the solution $v \neq 0$ is not localized near the wall; moreover Eq. (2.10) gives us the following solution:

$$u(x) = N [\text{sech}(x/\Delta)]^{m_0 \Delta}, \quad (2.12)$$

where N is a normalization constant, and $m_0 = g_Y \eta$ is the fermion mass in the broken phase. Finally, the expression for ψ is

$$\psi(x) = \frac{N}{\sqrt{2}} [\text{sech}(x/\Delta)]^{m_0 \Delta} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}. \quad (2.13)$$

The normalization constant N is bound to the surface charge density Q by

$$Q = \int_{-\infty}^{+\infty} dx j^0 = \int_{-\infty}^{+\infty} dx |\psi|^2, \quad (2.14)$$

and take the following value,

$$N^2 = \frac{Q}{\Delta B(m_0 \Delta, 1/2)}, \quad (2.15)$$

where $B(x, y)$ is the Bernoulli Beta Function.

3. Topological charged domain walls

In this Section we perform a more complete analysis by considering massless Dirac fermion coupled to a real scalar field trough the Yukawa coupling in presence of the electromagnetic field generated by fermions. Explicitly the dynamics is determined by the Lagrangian density

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4} (\phi^2 - \eta^2)^2 + \bar{\psi} (i \not{\partial} - g_Y \phi - e \not{A}) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (3.1)$$

Let us suppose that

$$\phi = \phi(x), \quad (3.2)$$

$$\psi = \psi(x, t) = \xi(x) e^{-i\omega t}, \quad (3.3)$$

$$A^\mu = A^\mu(x), \quad (3.4)$$

where ϕ , ξ and A^μ are real functions of x . We use the Lorentz gauge for the electromagnetic field, $\partial_\mu A^\mu = 0$. The Lagrangian density (3.1) with Eqs. (3.2)-(3.4) implies the equations of motion

$$\phi'' - \lambda (\phi^2 - \eta^2) \phi = g_Y \bar{\xi} \xi, \quad (3.5)$$

$$i\gamma^1 \xi' + (\omega\gamma^0 - g_Y \phi - eA) \xi = 0, \quad (3.6)$$

$$(A^\mu)'' = -e \bar{\xi} \gamma^\mu \xi. \quad (3.7)$$

With the ansatz (2.7), Eqs. (3.5)-(3.7) become

$$\phi'' + \lambda(\phi^2 - \eta^2) \phi = 0, \quad (3.8)$$

$$u' + g_Y \phi u = 0, \quad (3.9)$$

$$v' - g_Y \phi v = 0, \quad (3.10)$$

$$(A^0)'' = (A^2)'' = -2e(u^2 + v^2), \quad (3.11)$$

$$(A^1)'' = (A^3)'' = 0, \quad (3.12)$$

with $eA^0 = eA^2 + \omega$. Without loss of generality we can put $A^1 = A^3 = 0$. The equation (3.8) has been already analyzed in previous Section. The solutions of Eqs. (3.9) and (3.10) are given by Eq. (2.12) and $v = 0$ as observed in previous Section. Therefore the expression for ψ is

$$\psi(x, t) = \frac{N}{\sqrt{2}} [\text{sech}(x/\Delta)]^{m_0 \Delta} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-i\omega t}, \quad (3.13)$$

where N is given by Eq. (2.15). Taking into account Eq. (3.11) we obtain the electric and magnetic field $\mathbf{E} = (E_x, 0, 0)$, $\mathbf{B} = (0, 0, B_z)$, where

$$E_x = -(A^0)' = 2e \int_0^x dx' [u^2(x') + v^2(x')] = e \int_0^x dx' |\psi(x', t)|^2, \quad (3.14)$$

and $B_z = -E_x$. The electric field is perpendicular to the wall, while the magnetic field is parallel to the wall. They grow with x where the domain wall is at $x = 0$. The maximum value of the electric field is obtained when $x \rightarrow +\infty$, therefore

$$E_x^{(max)} = e \int_0^{+\infty} dx' |\psi(x', t)|^2 = \frac{eQ}{2}. \quad (3.15)$$

In the same way $B_z^{(max)} = eQ/2$. It is important to remark that this behavior is strictly connected to the non realistic case of infinite domain wall. Indeed, if we consider a disk of radius R in which a charge density eQ is stored, the value of the electric field is $E_x = (eQ/2)(1 - |x|/\sqrt{R^2 + x^2})$, where x is the distance from the disk: the electric field reduces when we depart from the disk. Therefore for finite domain walls, we expect that the electric and magnetic fields are null in the core of the defect, they reach the maximum value on the edge of the wall and then they decrease when we depart from the wall.

Let us consider a charged domain wall, that is a wall with fermion states localized on it. The surface energy density of such configuration is

$$\sigma_T = \sigma_w + \sigma_\psi + \sigma_E + \sigma_B, \quad (3.16)$$

where the density σ_w of the kink (2.9) is

$$\sigma_w = \frac{8}{3\lambda\Delta^3}, \quad (3.17)$$

the energy density referred to ψ is

$$\sigma_\psi = \omega Q, \quad (3.18)$$

while the energy densities associated to magnetic and electric fields are

$$\sigma_B = \sigma_E = \int_{-\infty}^{\infty} dx \frac{E_x^2}{2}. \quad (3.19)$$

In analogy with the case of a charged disk of finite radius, we can suppose that

$$\int_{-\infty}^{+\infty} dx E_x^2 \sim E_x^{(max)} l = e^2 Q^2 l, \quad (3.20)$$

with l is the linear dimension of the wall.

4. Non topological charged domain walls

In this Section we will consider the following physical system: a domain wall and a fermionic plane wave with his electromagnetic field in order to compare the energy density of such configuration with the physical case of the previous Section. Let us consider the equations of motion (3.5)-(3.7) in the case in which

$$\phi = \phi_{vac} + \delta\phi, \quad (4.1)$$

$$\psi = \psi_{vac} + \delta\psi, \quad (4.2)$$

$$A^\mu = A_{vac}^\mu + \delta A^\mu, \quad (4.3)$$

where the vacuum states of the system are

$$\phi_{vac} = \pm\eta, \quad \psi_{vac} = 0, \quad A_{vac}^\mu = 0. \quad (4.4)$$

In other words we write the field in terms of the fluctuations over the vacuum background. Under these assumptions, to the first order, the equations of motion in the linear approximation are

$$\partial_\mu \partial^\mu \delta\phi + 2\lambda \eta^2 \delta\phi = 0, \quad (4.5)$$

$$(i\gamma^\mu \partial_\mu - g_Y \phi_{vac}) \delta\psi = 0, \quad (4.6)$$

$$\partial_\mu \partial^\mu \delta A^\nu = e \delta\bar{\psi} \gamma^\nu \delta\psi. \quad (4.7)$$

Remembering that we have respectively $\phi_{vac} = +\eta$ for $x > 0$ and $\phi_{vac} = -\eta$ for $x < 0$, at fixed charge of configuration, the solution of Eqs. (4.5)-(4.7), that minimized the deviation of energy from its vacuum value, is

$$\delta\phi = 0, \quad (4.8)$$

$$\delta\psi_- = \frac{C}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{im_0 t} \quad \text{if } x < 0, \quad (4.9)$$

$$\delta\psi_+ = \frac{C}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} e^{im_0 t} \quad \text{if } x > 0, \quad (4.10)$$

$$\delta\mathbf{E} = (eC^2x, 0, 0), \quad \delta\mathbf{B} = 0, \quad (4.11)$$

where C is a constant. In the following, in order to avoid divergent values in the charge and energy calculation, we suppose that $\delta\psi$ and δE_x exist in large but finite segment of x axis: $-L/2 \leq x \leq L/2$. The constant C is related to the surface charge density by the following relation,

$$Q = \int_{-L/2}^{L/2} dx \, \delta\bar{\psi} \gamma^0 \delta\psi = \int_{-L/2}^0 dx \, |\delta\psi_-|^2 + \int_0^{L/2} dx \, |\delta\psi_+|^2 = C^2 L. \quad (4.12)$$

Considering the possibility of existence of stable topological charged solution, we have to compare the density energy σ_T of the solution having charge density Q with the sum of the energy of the kink (2.9) and the energy of charged non-topological configuration (4.8)-(4.11),

$$\sigma_{NT} = \sigma_w + \sigma_{\delta\psi} + \sigma_{\delta E}. \quad (4.13)$$

The energy density associated to $\delta\psi$ is

$$\sigma_{\delta\psi} = m_0 \int_{-L/2}^{L/2} dx \, |\delta\psi|^2 = m_0 Q, \quad (4.14)$$

while the electric energy density is

$$\sigma_{\delta E} = \int_{-L/2}^{L/2} dx \, \frac{\delta E_x^2}{2} \sim e^2 Q^2 L. \quad (4.15)$$

There is stable topological charged solution if

$$\sigma_T < \sigma_{NT}, \quad (4.16)$$

that is when

$$Q \lesssim \frac{\omega - m_0}{e^2(L-l)} \equiv Q^{crit}. \quad (4.17)$$

We observe that fermions from the broken phase (which have energy $E > m_0$) can be captured by the wall giving rise to a charged domain wall [¶]. Then when the charge density Q due to the fermionic mode solutions situated on the wall satisfy the inequality (4.17) we have that the topological structure of a CDW does not decay in domain wall and plane wave. In this way the existence of the CDW's is connected with the critical charge density Q^{crit} . The physical meaning of the critical charge density tell us that the first configuration is energetically favorable if $Q < Q^{crit}$, while if $Q > Q^{crit}$ we have a decomposition into a (kink)+(plane waves).

5. Conclusions and outlook

In this paper we have investigate the possible existence of charged domain walls. An important difference from Q-balls is the origin of the charge density Q . In our work the

[¶]We are supposing that the wall is finite but the linear dimension is much grater than the thickness of the wall. Moreover the integration along the axis x is performed over a distance L much greater than the linear dimension of the wall. So, we are working in the hypothesis that $L \gg l \gg \Delta$.

charge density does not derive from a complex scalar field but it is due to the presence of localized fermions on the wall.

We have analyzed the presence of an electric field (perpendicular to the wall) and magnetic field (parallel to the wall). It will be interesting to study as these fields might modify the interaction of the charged wall with the surrounded plasma and how the Coulomb barrier behaves in comparison with the absorption or repulsion with the plasma.

Moreover we have analyzed the stability of CDW's by confronting two physical situations: kink with localized fermions on the wall and (kink)+(fermion plane waves). The calculation of the energy density, for both, allowed to find a condition for the existence of CDW's with respect to the decomposition into a (kink)+(plane waves). We have found there is a critical value for the charge density, Q^{crit} : charged domain walls solutions exist when the charge density Q is smaller than Q^{crit} . In this case the charged domain walls survive, otherwise the system divided into a kink and plane waves, that is more favorable from an energetic point of view.

The fate of CDW's in the early Universe is determined by their lifetime and if these defects actually exist in the Universe, they may have interesting effects on the cosmology. An interesting hypothesis consists in to consider such objects as they would have survived until the present time and would contribute to the matter density of the Universe as a form of charged dark matter. This hypothesis has just been considered as regards the Q-balls, that are the ground state configurations for fixed charge Q in theories with interacting scalar fields that carry some global U(1) charge [6, 7]. It also will be interesting to understand the effect of CDW's on the structure formation in the early Universe.

However it must be pointed out that the existence of domain wall is still questionable. Indeed, in general the gravitational effects of just one such wall stretched cross the Universe would introduce a large anisotropy into the relic blackbody radiation. So that CDW's could have survived until today in the form of bubbles of radius R . In this case, it turns out that the bubbles get stabilized by the Coulomb repulsion. Indeed, the authors of Ref.[3] showed that these defects occur whenever the total charge is greater that a certain minimum value. In this case we are left with a gas of charged domain walls which, indeed, could be cosmological important.

Another important comment regards the connection between the primordial magnetic field and the magnetic field on the plane of a CDW. We can estimate its strength supposing that $B \sim B_z^{(max)} = eQ/2 \sim eQ^{crit}$. Taking, for example, $\omega = 2m_0$ and $L = 10l$ in Eq. (4.17) we have $Q^{crit} \sim m_0/l$, and for the magnetic field

$$B \sim \frac{g_Y \eta}{e l}. \quad (5.1)$$

Taking $g_Y/e \sim 1$ and $l = 10\Delta$ we obtain, at the electroweak phase transition ($v \sim 10^2 \text{ GeV}$), $B \sim 10^{22} \text{ Gauss}$. This value of the magnetic field is in accordance with the estimate obtained at the electroweak phase transition for primordial magnetic field [25]. Another consideration regards the possibility that CDW's can decay after the electroweak phase transition. They could protect baryons from the erasure of baryon number due to the sphaleron transitions, therefore they can also create the barionic asymmetry in the early Universe [26]. In any case, it is beyond the aim of present paper to examine these problems, which will be object of an upcoming work.

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